COOLING TIME OF CIRCULATING CRYOGENIC SYSTEMS

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The preliminary cooling time for cryogenic systems characterized by a high hydraulic resistance is calculated in the cases of laminar and turbulent modes of coolant flow.

In the development of circulating superconducting magnet systems (CSMS) and other cryogenic devices of great length (pipelines, superconducting electric power lines) it becomes necessary to estimate the time for cooling such systems to the working temperatures. In the present report such estimates are made for the cases of the absence and presence of an external inflow of heat.

A one-dimensional model was used in the calculations (the temperature is taken as constant over a cross section). In addition, the following assumptions are made.

1. The total heat exchange between the coolant and the cooled pipeline takes place in a section dx considerably shorter than the pipe length L.
2. Heat conduction in the longitudinal direction is absent.
3. As a consequence of assumptions 1 and 2 the temperature at the exit from the system remains constant and equal to the temperature of the surrounding medium during the entire cooling time.
4. The temperature at the entrance to the system ($x = 0$) at the time $t = 0$ becomes equal to the initial coolant temperature and remains constant to the end of the process.
5. The working temperature of the system is equal to the initial coolant temperature.

A diagram of the cooling process is presented in Fig. 1.

At any time $0 < t < t_{cool}$ the cooling pipeline can be represented as consisting of three sections: 1) a cooled section of length x; 2) a cooling section of length dx; 3) a warm section of length $L - (x + dx)$.

Let us set up the heat balance equation for the cooling section dx in the case of the absence of heat inflow from the surrounding medium

$$\bar{C}_m(T_2 - T_1)\,dM = m\bar{C}_p(T_2 - T_1)\,dt, \tag{1}$$

where $dM = \rho_m S dx$. Hence

$$dt = \frac{\bar{C}_m}{\bar{C}_p}\,\frac{\rho_m S}{m}\,dx.$$

Let us introduce the following notation:

$$\frac{\bar{C}_m}{\bar{C}_p} = \sigma; \quad \rho_m S = \frac{\rho_m SL}{L} = \frac{M}{L} = \rho\left[\frac{\text{kg}}{\text{m}}\right].$$

Then the cooling time is

$$t = \rho\sigma\int_0^L \frac{dx}{m}. \tag{2}$$

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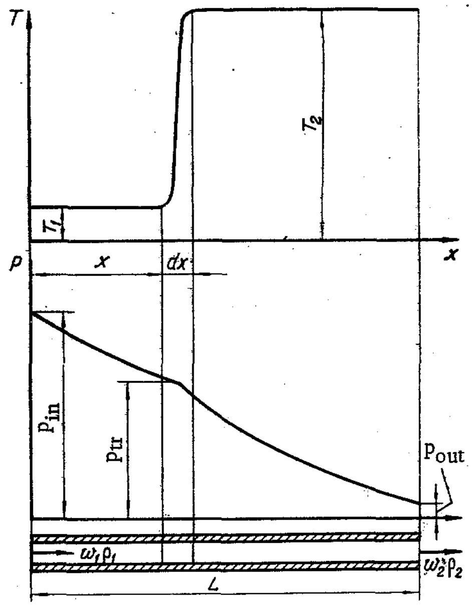

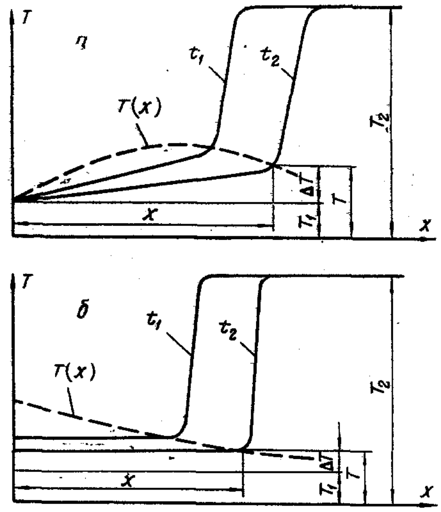

Fig. 1 Fig. 2

Fig. 1. Diagram of cooling process without allowance for heat inflow (T, °K; x, m; p, N/m²).

Fig. 2. Diagrams of cooling processes: a) in the presence of a heat inflow proportional to the length of the cooled section; b) in the presence of a heat inflow which is constant in time (T, °K; x, m).

We obtain the equation for the dependence of the mass flow rate on the length x from the equations for the hydraulic resistance in the cooled and warm sections, neglecting the hydraulic resistance in the cooling section dx:

$$\begin{cases} \Delta p_1 = p_{in} - p_{tr}, \\ \Delta p_2 = p_{tr} - p_{out}, \end{cases} \tag{3}$$

where

$$\Delta p = \frac{\rho w^2}{2} \lambda \frac{L}{d}.$$

In the case of a laminar mode of flow (Re ≤ 2300) the coefficient of resistance $\lambda_l = 64/\mathrm{Re}$. In the case of transitional or turbulent modes of flow (Re > 2300) we assume that the Blasius law is satisfied:

$$\lambda_t = \frac{0.3164}{\sqrt[4]{\mathrm{Re}}}.$$

Writing the equation for the Reynolds number in the form $\mathrm{Re} = md/\mu f$, for the laminar mode of flow we obtain

$$\begin{cases} p_{in} - p_{tr} = \dfrac{64 m \mu_1}{2\rho_1 f d^2} x, \\ p_{tr} - p_{out} = \dfrac{64 m \mu_2}{2\rho_2 f d^2} (L - x). \end{cases} \tag{4}$$

Assuming that the flow in the cold and warm sections is isothermal, we obtain the following equations for the mean flow densities:

$$\rho_1 = \frac{p_{in} + p_{tr}}{2RT_1}, \tag{5}$$

$$\rho_2 = \frac{p_{out} + p_{tr}}{2RT_2}. \tag{5'}$$

Substituting (5) and (5') into the system of equations (4), after transformations we obtain the equation

$$m = \frac{(p_{\text{in}}^2 - p_{\text{out}}^2) f d^2}{64R[\mu_1 T_1 x + \mu_2 T_2 (L - x)]} \quad (6)$$

Integrating (2) with allowance for (6), we determine the cooling time as

$$t_l = \frac{32\rho\sigma\mu_2 T_2 R L^2}{(p_{\text{in}}^2 - p_{\text{out}}^2) f d^2} \left[1 + \frac{\mu_1 T_1}{\mu_2 T_2}\right]. \quad (7)$$

We denote

$$32\mu_2 T_2 R = F_l, \quad 1 + \frac{\mu_1 T_1}{\mu_2 T_2} = \xi_l.$$

Then

$$t_l = \frac{F_l \rho}{f d^2}\, \sigma \frac{L^2}{(p_{\text{in}}^2 - p_{\text{out}}^2)}\, \xi_l. \quad (8)$$

In the case of the turbulent mode, after similar transformations we obtain

$$m = \frac{(p_{\text{in}}^2 - p_{\text{out}}^2)^{4/7} f d^{5/7}}{R^{4/7}\, 0.3164^{4/7}\, T_2^{4/7}\, \mu_2^{1/7}} \left[L - x\left(1 - \frac{T_1 \sqrt[4]{\mu_1}}{T_2 \sqrt[4]{\mu_2}}\right)\right] \quad (9)$$

and after integration of (2)

$$t_t = \frac{F_t \rho}{f d^{5/7}}\, \sigma \frac{L^{11/7}}{(p_{\text{in}}^2 - p_{\text{out}}^2)^{4/7}}\, \xi_t, \quad (10)$$

where

$$F_t = \frac{7}{11}(0.3164\, R T_2)^{4/7} \mu^{1/7}; \quad \xi_t = \frac{1 - \left(\frac{T_1}{T_2}\sqrt[4]{\frac{\mu_1}{\mu_2}}\right)^{11/7}}{1 - \frac{T_1}{T_2}\sqrt[4]{\frac{\mu_1}{\mu_2}}}.$$

It follows from Eqs. (8) and (10) that when the working temperature of the system is reduced with a constant pressure at the entrance its cooling time is shortened. This is explained, first, by the decrease in the hydraulic resistance of the cold section at a reduced working temperature, which leads to an increase in the flow rate of the coolant and consequently to a decrease in the cooling time. In Eqs. (8) and (10) this is expressed by a decrease in ξ_l and ξ_t with a decrease in the temperature T_1.

A second factor leading to a decrease in cooling time is a certain decrease in the mean heat capacity of the busbar in the temperature interval $T_2 - T_1$ with a decrease in T_1, while the mean heat capacity of the coolant remains almost unchanged in the same temperature interval, and therefore the specific flow rate σ of the coolant is reduced with a decrease in T_1.

Of course, a preliminary estimate of the mode of flow is necessary when using Eqs. (8) and (10).

In the derivation of Eqs. (8) and (10) it was assumed that heat inflow from the surrounding medium is absent. However, the effect of heat inflow on the duration of the cooling can prove to be very significant.

A strict determination of the cooling time of the system with allowance for the heat inflow is rather difficult, first because of the complexity of the analytical solution of the differential equation

$$m\bar{C}_p (T_2 - T)\, dt = \rho \bar{C}_m (T_2 - T)\, dx + Q(x;\ T)\, dT,$$

and second because an estimate of the size of the heat inflow itself and its distribution law can be made only in rough approximations. Nevertheless, it is useful to find the solution for the simplest types of heat inflow, mainly to estimate the applicability of Eqs. (8) and (10) in the presence of a heat inflow.

The most characteristic for linear systems of great length (pipelines, superconducting electric power lines, etc.) is a heat inflow proportional to the length of the cooled section (Fig. 2a): $Q = qx$ (q is in W/m).

We assume that the specific heat inflow is constant along the length of the cooled section.

In this case the heat-balance equation for the cooling section is

$$m^*\bar{C}_p(T_2-T)\,dt = \bar{C}_m^*\rho(T_2-T)\,dx. \tag{11}$$

Hence,

$$dt = \frac{\bar{C}_m^*\rho}{\bar{C}_p m^*}\,dx,$$

where

$$\bar{C}_m^* = \frac{1}{T_2-T}\int_T^{T_2} C_m\,dT, \tag{12}$$

$$m^* = m^*(x;\ T).$$

For the cooled section

$$[m^*C_p(T-T_1)-qx]\,dt - C_m(T)\rho x\,dT = 0.$$

With moderate heat inflows and a low coolant temperature the term $C_m(T)\rho x dT$ can be neglected, and then

$$T = T_1 + \frac{qx}{m^*C_p}.$$

The presence of a heat inflow leads to an increase in the coolant temperature $T = T_1 + \Delta T$ in front of the cooling section and to an increase in the temperature of the cooled section (which we assume to be the same along the length of the cooled section at each moment and equal to $T_{av} = T_1 + \Delta T/2$). This, in turn, causes an increase in the mean heat capacity $\bar{C}_m^*(T)$ and a decrease in the flow rate of the coolant, which finally results in an increase in the duration of the cooling.

In Figs. 3 and 4 we present $\sigma^* = \bar{C}_m^*/\bar{C}_p$ and $(1/\bar{m}^*)$ as functions of the temperature.

It is clear that the variation in the value $(1/\bar{m}^*)$ as a function of the temperature in the temperature interval under consideration (20–80°K) can be neglected.

In the determination of the cooling time we assume that the coolant temperature in the cooled sections is constant during the cooling process and equal to $T = T_1 + \Delta\bar{T}$, where

$$\Delta\bar{T} = \frac{q}{C_pL}\int_0^L \frac{x}{m^*}\,dx \approx \frac{q}{C_pL}\int_0^L \frac{x}{m}\,dx,$$

and if

$$\mu_2T_2 \gg \mu_1T_1, \quad \overline{\Delta T} = \frac{32\mu_2T_2RL^2q}{3(p_{in}^2-p_{out}^2)fd^2C_p}.$$

Hence,

$$t_{ql} = \frac{F_l\rho}{fd^2}\sigma^*\frac{L^2}{p_{in}^2-p_{out}^2}\xi_l^* = t_l\frac{\sigma^*\xi_l^*}{\sigma\xi_l}. \tag{13}$$

For the turbulent mode of flow with all the same assumptions we obtain

$$\bar{T} = T_1 + \frac{qL}{4f\left[\dfrac{(p_{in}^2-p_{out}^2)\,d}{0.3164\,RT_2L}\right]^{4/7}\left(\dfrac{d}{\mu_2}\right)^{1/7}C_p},$$

$$t_{qt} = \frac{F_t\,\rho\sigma^*L^{11/7}}{fd^{5/7}(p_{in}^2-p_{out}^2)^{4/7}}\xi_t^* = t_t\frac{\sigma^*\xi_t^*}{\sigma\xi_t}. \tag{14}$$

Equations (12) and (13) can be used with a sufficient degree of accuracy in the case when $T \le 70$–80°K.

Let us examine another particular case, when the heat inflow can be considered as independent of the length of the cooled section and constant in time (Fig. 2b).

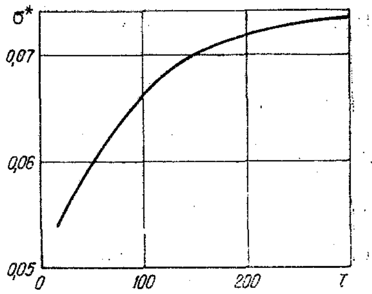

Fig. 3

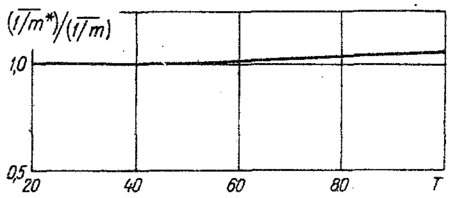

Fig. 4

Fig. 3. Ratio of mean heat capacity of copper to mean heat capacity of helium as a function of the helium temperature in front of the cooling section [$\rho^*(T)$ is a dimensionless value].

Fig. 4. Variation in the reduced value $(1/\bar{m}^*)/(1/\bar{m})$ as a function of the temperature T, °K.

The heat-balance equation for the section dx in this case is analogous to Eq. (11).

The coolant temperature in front of the cooling section as a function of the length of the cooled section is

$$T = T_1 + \frac{Q}{m^* C_p}, \quad m^* = m^*(T;\ x),$$

in the laminar mode of flow

$$\bar{T} = T_1 + \frac{32R\mu_2 LQ}{C_p(p_{in}^2 - p_{out}^2) f d^2},$$

and in the turbulent mode of flow

$$\bar{T} = T_1 + \frac{7Q}{11f\left[\frac{(p_{in}^2 - p_{out}^2)d}{0.3164\,RLT_2}\right]^{4/7}\left(\frac{d}{\mu_2}\right)^{1/7} C_p}.$$

The mean heat capacity $\bar{C}^*$ of the system and the cooling time for the different modes of flow (t_{ql}, t_{qt}) are determined from Eqs. (12), (13), and (14).

The instrumentation of a test stand designed for the testing of large superconducting magnet systems (SMS), including circulating SMS, was used for the experimental determination of the preliminary cooling time.

The time for the cooling of circulating SMS from room temperature to 20°K was measured in the experiments. The current-carrying element had nine channels for the flow of the coolant, each with a nominal diameter of 2 mm.

In the first tests the SMS consisted of two hydraulically parallel branches each 200 m long. Later systems with branches each 400 m long were tested.

The cooled helium was supplied to the system through a helium pipeline from a neon–helium liquefier. We tried to keep the temperature and pressure of the gas at the entrance to the system constant during an experiment. The values of these parameters lay in the following ranges for different tests: entrance gauge pressure from 10 to 25 atm, entrance temperature 20–30°K.

The pressure was measured with a standard manometer. Allen-Bradley carbon resistance thermometers, in which the voltage drop was determined with a P-309 potentiometer, were used as the pickups in the temperature measurement.

The course of the cooling process was observed from the variation in the electrical resistance of the system.

The coefficient of hydraulic resistance of the system was somewhat higher than that determined from the Blasius equation. This is evidently explained by the presence of local resistances and the

TABLE 1. Experimental and Calculated Values

L, m	p_{in}, atm	T_1	Q, W	$\lambda Re^{1/4}$	t_t, h	t_{qt}, h	t_{exp}, h	Δ, %
200	8,9	40	40	0,44	7,47	9,00	12,0	29
400	22,0	40	100	0,42	6,53	7,83	9,5	17,9
400	15,5	40	20	0,42	9,4	9,9	11,0	14,5
400	15,5	40	30	0,43	,4	10,0	11,5	13

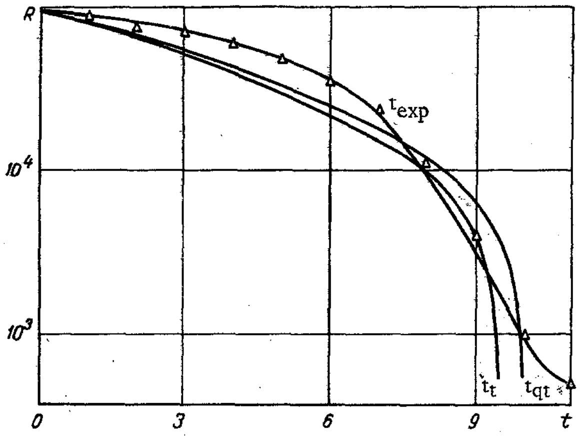

Fig. 5. Comparison of experimental and calculated curves of variation in electrical resistance R in μΩ of the system as a function of the time t, h.

hydraulic resistance of the input and output pipelines which was not taken into account in the model. The constant in the Blasius equation was determined experimentally for the calculations.

The experimental and calculated (both without allowance for heat inflow and with such allowance) curves of the variation in the electrical resistance of the system as a function of the time (the dynamics of the cooling process) are compared in Fig. 5. For the curves presented

$$p_{in.av} = 15.5 \cdot 10^5\ N/m^2;\ T_{1av} = 40°K;$$
$$Q = 20\ W;\ \lambda Re^{1/4} = 0.42.$$

A comparison of the experimental and calculated data for different experiments is presented in Table 1.

A certain excess in the actual cooling time over the calculated time is connected with the fact that the real cooling process differs from the calculated model by the presence of a finite heat conduction between the coils in the radial and axial directions.

This results in the incomplete utilization of the coolant, reducing its temperature at the exit from the system. The disagreement between the experimental and calculated values can be partially explained by the ignoring of external heat inflow to the system in the comparison.

Nevertheless, as seen from Table 1 the results of the experiment are in satisfactory agreement with the calculated dependences, which permits one to recommend the latter for the estimate of the cooling time of circulating SMS.

NOTATION

w is the flow velocity, m / sec;

M is the mass being cooled, kg;

ρ_m is the density of material being cooled, kg / m³;

$\bar{C}_m = \frac{1}{\Delta T}\int_{T_1}^{T_2} C_m dT$ is the mean heat capacity of material being cooled in the temperature interval T_1, T_2, J / kg · deg;

$\bar{C}_p = \frac{1}{\Delta T}\int_{T_1}^{T_2} C_p dT \approx C_p$ is the mean heat capacity of coolant in the temperature interval T_1, T_2, J / kg · deg;

T_1 is the coolant temperature, °K;

T_2 is the temperature of surrounding medium, °K;

$\rho_{1,2}$ is the mean density of coolant in cold and warm sections, kg / m³;

p_{in} is the pressure at entrance to system, N / m²;

p_{out} is the pressure at exit from system, N / m²;

p_{tr} is the transition pressure, N / m²;

f is the cross-sectional area of channel, m²;

d is the channel diameter, m;

$\mu_{1,2}$ is the dynamic viscosity of coolant in cold and warm sections, kg / m · sec;

S is the cross-sectional area of body being cooled, m²;

m	is the flow rate of coolant, kg / sec;
R	is the gas constant, J / kg · deg;
L	is the length of system, m;
t	is the coolant time, sec;
Re	is the Reynolds number;
q	is the specific heat inflow per linear meter, W/m;
Q	is the heat inflow, W;
λ	is the coefficient of hydraulic resistance in Blasius equation.